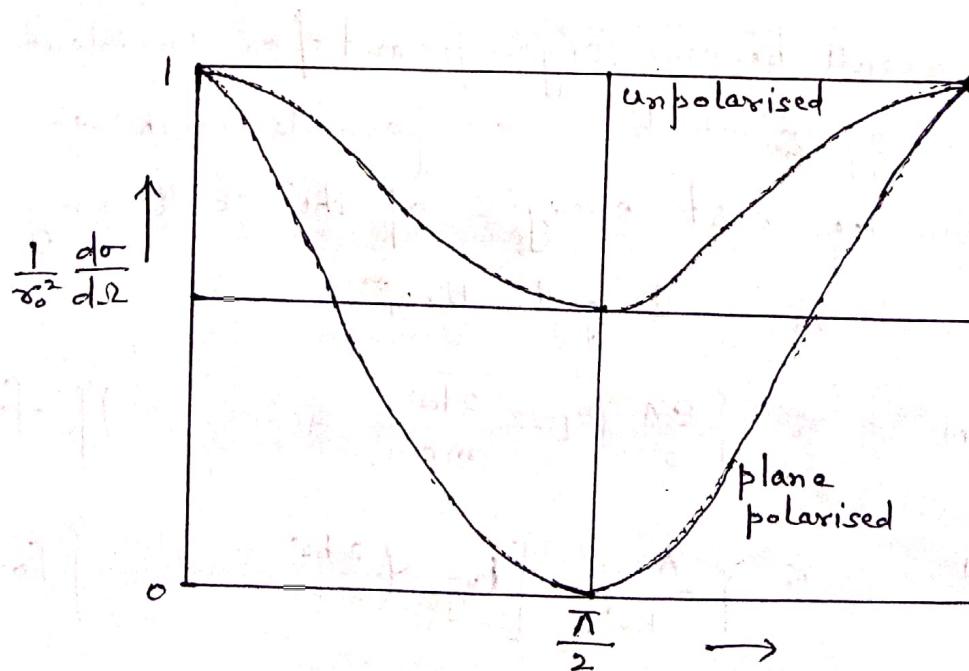


In it angle  $\phi$  is called the scattering angle and the factor  $\frac{1}{2}(1 + \cos^2\phi)$  is called the degree of polarisation. From expression (B) it is clear that

- (i) scattering of electromagnetic waves is independent of the nature of incident wave, (i.e.).
- (ii) scattering occurs in all directions and is maximum when  $\phi = 0$  or  $\pi$  (i.e. in the forward and backward directions) while minimum when  $\phi = \frac{\pi}{2}$  or  $\frac{3\pi}{2}$  (i.e. in the side way directions).
- (iii) scattering depends on the charged particle i.e scattered and is symmetrical about the line given by  $\phi = \pi/2$ .

Fig. is a plot of differential scattering cross section as a function of scattering angle  $\phi$  for plane polarised and unpolarised incident radiations.



Total scattering cross section will be

$$\begin{aligned}
 \sigma &= \int \frac{d\sigma}{d\Omega} d\Omega^* \\
 &= \int r_0^2 \frac{1}{2} (1 + \cos^2 \phi) d\Omega \\
 &= r_0^2 \int_0^\pi \frac{1}{2} (1 + \cos^2 \phi) 2\pi \sin \phi d\phi \\
 &\quad [as d\Omega = 2\pi (1 - \cos \phi)] \\
 &= r_0^2 \pi \int_0^\pi (\sin \phi + \cos^2 \phi \sin \phi) d\phi \\
 &= r_0^2 \pi \left[ -\cos \phi - \frac{\cos^3 \phi}{3} \right]_0^\pi \\
 \sigma_t &= \frac{8\pi}{3} r_0^2
 \end{aligned}$$

Result (C) was first of all derived by Thomson and so after his name it is called Thomson scattering cross section.

A quantum mechanical calculation carried out by Kelvin and Nishina shows that deviations from Thomson's result become significant for incident photon energy  $\frac{h\nu}{m_e c^2}$  which is comparable with or larger than the rest energies of the scattering electron  $m_e c^2$ . According to them

$$P_{KN} = \gamma_0^2 \left\{ \frac{8\pi}{3} \left( 1 - \frac{2\hbar\omega}{mc^2} + \dots \right) \right\} \text{ for } \hbar\omega \ll mc^2$$

and o-KN

$$= \gamma_0^2 \left\{ \frac{\pi m c^2}{\hbar \omega} \left[ \log_e \left( \frac{2\hbar\omega}{mc^2} + \frac{1}{2} \right) \right] \right\} \text{ for } \hbar\omega \gg mc^2$$

These results are shown in the fig.

If the incident radiations are plane polarised  
then

$$\sigma = \int \gamma_0^2 \left\{ \frac{\pi mc^2}{hv} \left[ \log \left( \frac{2hv}{mc^2} + \frac{1}{2} \right) \right] \right\}$$

$$= \int \gamma_0^2 \sin^2 \theta d\Omega$$

$$= \int_{-\pi/2}^{\pi/2} \gamma_0^2 \sin^2 \theta 2\pi \cos \theta d\theta$$

$$\text{ie } \sigma = 4\pi \gamma_0^2 \int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta$$

$$= 4\pi \gamma_0^2 \left[ \frac{\sin^2 \theta}{3} \right]_0^{\pi/2}$$

$$= \frac{4}{3}\pi \gamma_0^2$$

$$\sigma = \int_0^{\pi} \gamma_0^2 \cos^2 \phi d\phi \quad [\text{as } (\sin^2 \theta = \cos^2 \phi)]$$

$$\text{ie } \sigma = \int_0^{\pi} \cos^2 \phi 2\pi \sin \phi d\phi = 2\pi \gamma_0^2 \int_0^{\pi} \cos^2 \phi \sin \phi d\phi$$

$$= 2\pi \gamma_0^2 \left[ -\frac{\cos^2 \phi}{3} \right]_0^{\pi} = \frac{4}{3}\pi \gamma_0^2$$

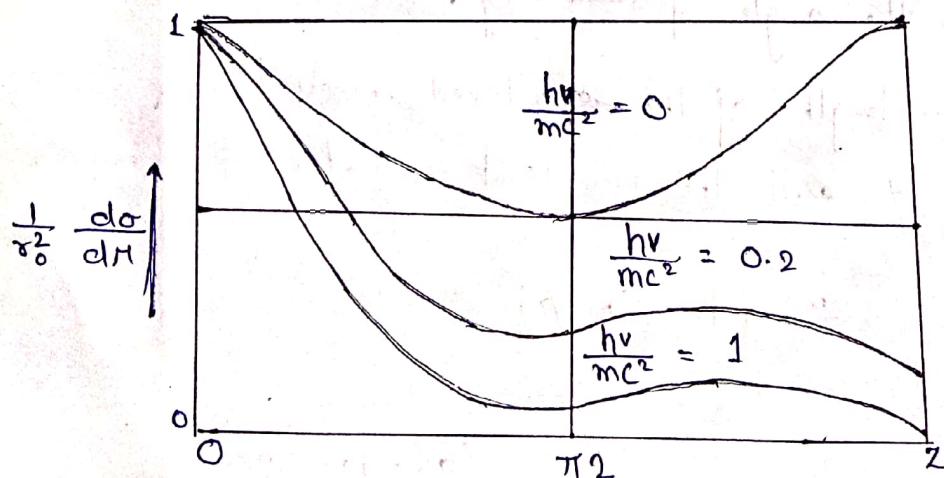


Fig. 7.4.

From these curves it is clear that

- (i) the scattering depends on the nature of incident radiations.
- (ii) Quantum mechanical result approaches the classical one on the long wavelength side the frequency  $\omega = \frac{\omega}{2\pi}$  goes to zero.
- (iii) The scattering is not symmetrical. In general the scattered radiation is more concentrated in the forward direction i.e.  $\phi = 0$ .

A part from these there is another feature of Thomson scattering which is modified by quantum considerations. Classically the scattered radiation has the same frequency as the incoming waves but quantum mechanical calculation shows that the frequency of the scattered radiation is lesser than that of incoming waves and depends on the angle of scattering. The relation between the wavelength of the scattered radiation is lesser than that of incoming waves and depends on the angle of scattering. The relation between the wavelength of the scattered radiation at an angle of  $\phi$  and the incident radiation is

$$\lambda_s = \lambda_i + \frac{h}{mc} (1 - \cos \phi)$$

— X —

(06-10-2020)